$$+k^3\left(\qquad\qquad\qquad\qquad\qquad\qquad + \frac{7}{20}\, m^6\right)\Big\}$$

$$+ \cos 6\,\mathrm{D} \Big\{ \; k^3\left(\qquad\qquad\qquad\qquad\qquad\qquad - \frac{59}{256}\, m^6\right)\Big\}.$$

and

$$v = t$$

$$+ \sin 2\,\mathrm{D} \Big\{ \; k\left(\frac{11}{8}m^2 + \frac{59}{12}m^3 + \frac{929}{72}m^4 + \frac{896}{27}m^5 + \frac{41975}{048}\, m^6\right)$$

$$+k^2\left(\qquad\qquad - \frac{1}{2}\, m^4 - \frac{41}{12}\, m^5 - \frac{43}{3}\, m^6\right)$$

$$+k^3\left(\qquad\qquad\qquad\qquad\qquad\qquad - \frac{783}{2048}\, m^6\right)\Big\}$$

$$+ \sin 4\,\mathrm{D} \Big\{ \; k^2\left(\qquad\qquad \frac{201}{256}m^4 + \frac{649}{120}m^5 + \frac{665263}{28800}m^6\right)$$

$$+k^3\left(\qquad\qquad\qquad\qquad\qquad\qquad - \frac{49}{80}\, m^6\right)\Big\}$$

$$+ \sin 6\,\mathrm{D} \Big\{ \; k^3\left(\qquad\qquad\qquad\qquad\qquad\qquad + \frac{3715}{6144}\, m^6\right)\Big\};$$

which for $k = 1$ agree with the foregoing formulæ (verifying them as far as m^5); the present formulæ exhibit the manner in which the expressions depend on the several powers of the disturbing force.

On the Variations of the Position of the Orbit in the Planetary Theory. By Prof. Cayley.

It has always appeared to me that in the Planetary Theory, more especially when the method of the variation of the elements is made use of, there is a difficulty as to the proper mode of dealing with the inclinations and longitudes of the nodes, hindering the ulterior development of the theory. Considering the case of two planets m, m', and referring their orbits to any fixed plane and fixed origin of longitudes therein, let θ, θ' be the longitudes of the nodes, ϕ, ϕ' the inclinations ($p = \tan\phi \sin\theta$, $q = \tan\phi\cos\theta$, &c., as usual); then the disturbing functions for m, m' respectively are developed, not explicitly in terms of ϕ, ϕ', θ, θ', but in terms of Φ, the mutual inclination of the two orbits, and of Θ, Θ' the longitudes in the two orbits respectively of the mutual node of the two orbits; Φ and Θ, Θ' being functions (and complicated

ones) of $\phi, \phi', \theta, \theta'$. Moreover, although in the general theory of the secular variations of the orbits of the planetary system, θ, ϕ, &c., are, as above, referred to one fixed plane (the ecliptic of a certain date), yet in the theory of each particular planet it is the practice, and obviously the convenient one, to refer for such planet the θ, ϕ to its own fixed plane (the orbit of the planet at a certain date), the effect of course being that ϕ, and consequently p, q, instead of being of the order of the inclinations to the ecliptic, are only of the order of the disturbing forces. It has occurred to me that the last-mentioned plan should be adhered to *throughout;* viz., that for each planet m, the position of its variable orbit should be determined by θ, the longitude of its node, and ϕ, the inclination in reference to the appropriate fixed plane (orbit of the planet at a certain date) and origin of longitude therein. The disturbing functions for the planet m and m' will of course depend not only on $\theta, \theta', \phi, \phi'$, but on the quantities Φ, Θ, Θ' which determine the mutual positions of the two fixed planes of reference and origins of longitude therein, *these last being however absolute constants not affected by any variation of the elements;* so that as regards the variation of the elements the disturbing functions are in fact given as *explicit* functions of the variable elements $\theta, \theta', \phi, \phi'$; and where ϕ, ϕ' and therefore also p, q, p', q' are only of the order of the disturbing forces.

I proceed to work out this idea, for the present considering the development of the Disturbing Function only as far as the first powers of p, q &c. For comparison with the ordinary theory, observe that in this theory the disturbing function contains only the *second* powers of the p, q &c., made use of therein; these are in fact of a form such as $\mathrm{P} + p$, $\mathrm{Q} + q$, . . . where P, Q are absolute constants and p, q . . . are the p, q, . . . of the present theory; the ordinary theory gives therefore in the disturbing function a series of terms involving $(\mathrm{P}+p)^2$, $(\mathrm{P}+p)(\mathrm{Q}+q)$, ... which I now take account of only as far as the first powers of p, q, . . . viz., they are in effect reduced to $\mathrm{P}^2 + 2\mathrm{P}p$, $\mathrm{PQ} + \mathrm{P}q + \mathrm{Q}p$, &c. . . . The present theory is thus not now developed to the extent of giving the p, q . . . of the ordinary theory in the more complete form as the solutions of a system of simultaneous linear differential equations, but only to the extent of obtaining for these p, q . . . respectively the terms which are proportional to the time.

I commence with the following subsidiary problem. Consider a spherical triangle A B C (sides a, b, c, angles A, B, C, as usual), and taking the side c as constant, but the angles A and B as variable, let it be required to find the variations of C, a, b in terms of variations $d\mathrm{A}$, $d\mathrm{B}$ and the variable elements C, a, b themselves. Although the geometrical proof would be more simple, I give the analytical one, as it may be useful.

We have

$$\cos \mathrm{C} = -\cos \mathrm{A} \cos \mathrm{B} + \sin \mathrm{A} \sin \mathrm{B} \cos c,$$

and thence

$$-\sin C\, dC = (\sin A \cos B + \cos A \sin B \cos c)\, dA$$
$$+ (\sin B \cos A + \sin A \cos B \cos c)\, dB$$
$$= \frac{\sin B \sin c}{\tan b}\, dA + \frac{\sin A \sin c}{\tan a}\, dB,$$

that is

$$-\frac{\sin C}{\sin c}\, dC = \frac{\sin B \cos b}{\sin b}\, dA + \frac{\sin A \cos a}{\sin a}\, dB,$$

or finally,

$$-dC = \cos b\, dA + \cos a\, dB.$$

Next

$$\sin a = \sin c \frac{\sin A}{\sin C},$$

or differentiating

$$\cos a\, da = \frac{\sin c}{\sin^2 C}(\sin C \cos A\, dA - \cos C \sin A\, dC),$$

or substituting for dC its value,

$$= \frac{\sin c}{\sin^2 C}\left\{ dA (\sin C \cos A + \cos C \sin A \cos b) + dB \,.\, \cos C \sin A \cos a \right\}$$

$$= \frac{\sin c}{\sin^2 C}\left\{ dA \frac{\sin A \sin b \cos a}{\sin a} + dB \cos C \sin A \cos a \right\},$$

hat is,

$$da = \frac{1}{\sin C}\left\{ dA \frac{\sin A}{\sin a} \sin b + dB \cos C \sin A \right\} \div \frac{\sin C}{\sin c},$$

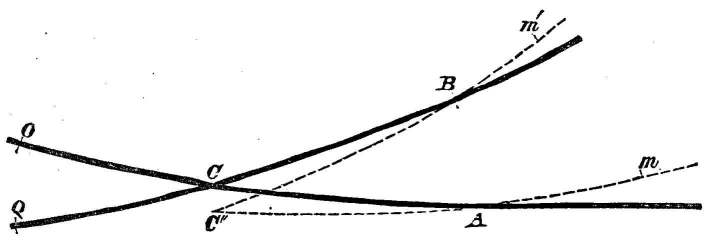
or on the right-hand writing $\frac{\sin A}{\sin a}$ instead of $\frac{\sin C}{\sin c}$ this is

$$da = \frac{1}{\sin C}(dA \sin b + dB \cos C \sin a).$$

And similarly,

$$db = \frac{1}{\sin C}(dB \sin a + dA \cos C \sin b).$$

Now let the continuous lines represent the orbits of m, m' at certain dates, O, Q the origins of longitude therein; and the



dotted lines the variable orbits of the planets respectively.

Write

$$\begin{aligned} &OC = \Theta,\ CA = \theta,\ \angle\, CAC' = \phi, \\ &QC = \Theta',\ CB = \theta',\ \angle\, CBC = \phi', \\ &\angle\, C = \Phi. \end{aligned}$$

Then, answering to the rotation of the lemma, we have

$$a = \theta',\quad b = \theta,\quad C = \Phi, \qquad dA = \phi,\quad dB = -\phi'$$
$$\text{or say} \qquad = \tan\phi, \qquad = -\tan\phi',$$

whence

$$\begin{aligned} C'B &= a + da \\ &= \theta' + \frac{1}{\sin\Phi}(\tan\phi\sin\theta - \tan\phi'\cos\Phi\sin\theta') \\ &= \theta' + \frac{1}{\sin\Phi}(p - p'\cos\Phi), \\ C'A &= b + db \\ &= \theta + \frac{1}{\sin\Phi}(-\tan\theta'\sin\phi' + \tan\phi\cos\Phi\sin\theta) \\ &= \theta - \frac{1}{\sin\Phi}(p' - p\cos\Phi), \\ \angle\, C' &= C + dC = \Phi - \cos\theta\tan\phi + \cos\theta'\tan\phi' \\ &= \Phi - q + q'. \end{aligned}$$

Suppose v, v' are the longitudes of the planets in their two orbits respectively; that is,

$$\begin{aligned} &v = OA + Am = \Theta + \theta + Am, \\ &v' + QB + Bm' = \Theta + \theta' + Bm', \end{aligned}$$

whence

$$C'm = C'A + Am = v - \Theta - \frac{1}{\sin\Phi}(p' - p\cos\Phi),$$

$$C'm' = C'B + Bm = v' - \Theta' + \frac{1}{\sin\Phi}(p - p'\cos\Phi),$$

$$\angle C' = \qquad \Phi - q + q';$$

say these values are $v - \Theta + x$, $v' - \Theta' + x'$, $\Phi + y$. Then if $\overline{H}$ is the angular distance $m\,m'$ of the two planets,

$$\begin{aligned}
\cos\overline{H} &= \cos(v - \Theta + x)\cos(v' - \Theta' + x') \\
&\qquad + \sin(v - \Theta + x)(\sin v' - \Theta' + x') \,.\, \cos(\Phi + y) \\
&= \cos(v - \Theta)\cos(v' - \Theta') + \sin(v - \Theta)\sin(v' - \Theta')\cos\Phi \\
&\quad + x\,[-\sin\overline{v - \Theta}\cos\overline{v' - \Theta'} + \cos\overline{v - \Theta}\sin\overline{v' - \Theta'}\cos\Phi] \\
&\quad + x'[-\cos\overline{v - \Theta}\sin\overline{v' - \Theta'} + \sin\overline{v - \Theta}\cos\overline{v' - \Theta'}\cos\Phi] \\
&\quad + y\,[-\sin\overline{v - \Theta}\sin\overline{v' - \Theta'}\sin\Phi]. \\
&= \cos H + \nabla \text{ suppose.}
\end{aligned}$$

The disturbing function for the planet m disturbed by m' is

$$\Omega = m'\left\{\frac{1}{\sqrt{r^2 + r'^2 - 2\,r\,r'\cos\overline{H}}} - \frac{r\cos\overline{H}}{r'^2}\right\}.$$

($\Omega = -R$, if R is the disturbing function of the *Mécanique Céleste*); and the term hereof which involves ∇ is

$$\nabla\frac{d\,\Omega}{d\,.\,\cos\overline{H}}$$

where after the differentiation $\cos\overline{H}$ is replaced by $\cos H$

$$= m'\left\{\frac{r\,r'}{(r^2 + r'^2 - 2\,r\,r'\cos H)^{\frac{3}{2}}} + \frac{r}{r'^2}\right\}\nabla$$

viz., this a linear function of x, x', y, that is of p, q, p', q', with coefficients which of course involve the other variable elements and the time; but it will be remembered that Θ, Θ', Φ are not variable elements, but are absolute constants. The variations of p depend upon $\frac{d\,\Omega}{d\,q}$ and those of q on $\frac{d\,\Omega}{d\,p}$, and the quantities p, q, p', q' . . . disappear from these differential coefficients $\frac{d\,\Omega}{d\,q}$, $\frac{d\,\Omega}{d\,p}$; that is, disregarding periodic terms, and the variations of the elements, we obtain $\frac{dp}{dt}$, $\frac{d\,q}{d\,t}$ as absolute constants, or reckoning the time from the epoch belonging to the fixed orbit of m, we

have p, q as mere multiples of the time ($p = \mathrm{A}\,t$, $q = \mathrm{B}\,t$, where A and B are constants); agreeing with the statement preceding the investigation.

Observe that the p, q, as used above, have reference not only to the fixed orbit of m, but also to the node thereon of the fixed orbit of m': we may, if we please write, $\mathrm{p} = \tan\phi \sin(\Theta + \theta)$, $\mathrm{q} = \tan\phi\cos(\Theta + \theta)$, that is, $\mathrm{p} = q \sin \Theta + p \cos \Theta$, $\mathrm{Q} = q \cos \Theta - p \sin \Theta$, (or $p = \mathrm{p} \cos \Theta - \mathrm{q} \sin \Theta$, $q = \mathrm{P} \sin \Theta + \mathrm{q} \cos \Theta$) and in place of p, q introduce into the formulæ p and q, which have reference only to the fixed orbit of m, and similarly writing $\mathrm{p}' - \tan\phi' \sin(\Theta' + \theta')$, $\mathrm{q}' = \tan\phi' \cos(\Theta + \theta')$, instead of p', q' introduce p', q' which have reference only to the fixed orbit of m'.

I remark that a table for the relative positions of the orbits of the eight Planets for the Epoch 1st January, 1850, is given in Leverrier's *Annales de l'Observ. de Paris*, t. ii. (1856), pp. 64–66.

On Uniformity in the Measurement of Position-Angles with the Telescope. By Capt. William Noble.

Inasmuch as no record exists of the *vivâ voce* discussions which take place at our meetings, I am induced to put into a more permanent form an oral suggestion which I made, on the spur of the moment, during a debate which arose on the night of January 12th last. My suggestion then was this: that one uniform system of measuring position-angles in the field of a telescope should be adopted. Every double-star observer in the world, without exception, measures the angles made with the

Fig. 1

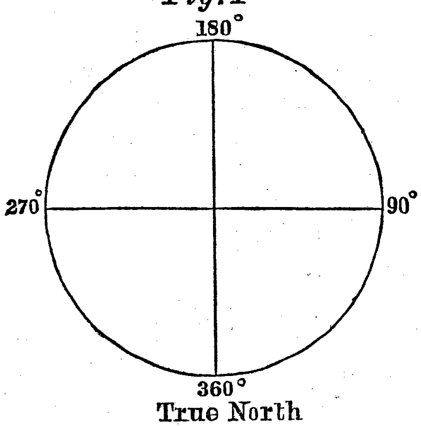
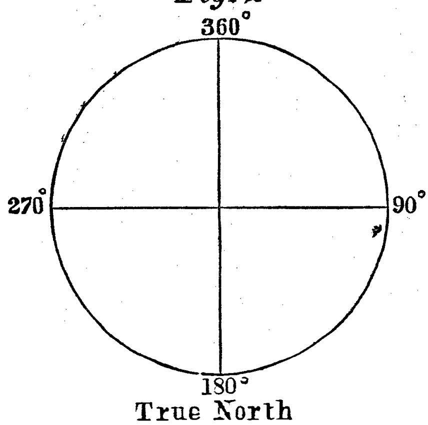


Fig. 2



meridian by a line joining the two components of the system, in the direction indicated in fig. 1; in which (assuming the ordinary Huyghenian or the Ramsden eye-piece to be employed) the bottom of the field is, of course, the true north. This, then, is, I may pretty confidently assert, the mode and direction in which any regular observer with the parallel-wire position-micrometer would naturally regard a circle, concentric with his visual telescopic field, to be divided. When, however, we come to study

the predictions of Occultations of Stars by the Moon given in the *Nautical Almanac*, we find the angles at which disappearance and reappearance are computed to take place no longer reckoned from the north round by east, south, and west, but having their initial point at the south, and being measured round by east, north, and west, as in fig. 2.

The reason for this I have utterly failed to discover. It seems to me to introduce a wholly needless complication, and one particularly embarrassing to the student and young astronomer; and I would therefore venture to submit that the mode of reckoning angles in the case of lunar occultations should be assimilated to that employed for measuring the position-angles of double stars. This need not, of course, prevent a supplementary calculation of the angle from the Moon's vertex, if thought necessary or desirable; but even this I would have measured round by the west, and not by the east, so as still to preserve the identity of the two systems.

Forest Lodge, Maresfield, March 2, 1872.

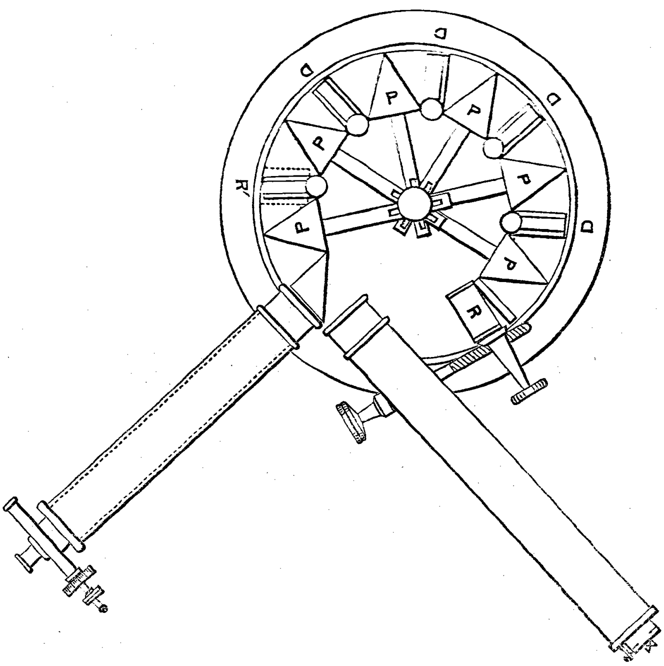
Note on an unsuspected Cause of Diffraction Phenomena in a Telescope. By Capt. William Noble.

Some little time ago, in observing *Jupiter* and his satellites, I remarked certain emanations which appeared to have their origin in diffraction. I was very much puzzled to imagine in what way these phenomena could arise. My 4·2-inch Ross object-glass is simply perfect; my eyepieces were carefully cleaned, and, so far as I could see by removing them, and looking up the tube at the Moon or *Jupiter*, the tube itself was free from any obstruction. Since, however, the general definition of the instrument was sensibly unimpaired, I took no further action in the matter, and let things take their course until about the middle of last month. I was observing the Sun one morning at that period, when I removed the eyepiece for some reason, and happened to glance obliquely up the tube. To my astonishment I saw, brilliantly illuminated by the Sun, a perfect *grating* of excessively fine spider-webs, spun vertically across the interior of the telescope, somewhat within the focus of the object-glass. A light, in more senses than one, suddenly broke in upon me, and I very speedily removed the offending lines. I had the pleasure, that same evening, of viewing the Jovian system shorn of all optical appendages. I am too ignorant of the Arachnida to be able to guess at the species which produces a web of such extraordinary tenuity; but it certainly must be an extremely minute one, not only on account of the excessive fineness of the filaments which it spins, but also in order that it should have found its way inside of a tube so thoroughly and carefully closed as (I should think) to prevent the existence of interstice or aperture whatever whereby an entry might be effected.

Forest Lodge, Maresfield, March 2, 1872.

On an Universal Automatic Spectroscope.
By John Browning.

The diagram represents my automatic spectroscope, so made that any number of prisms from 1 to 6 may be used at pleasure; and as the ray is reversed, the power thus gained will vary from 2 to 12 prisms. This is achieved by so adjusting the right-angled prism that it may be slid into dovetails between each of the prisms, and the ray thus turned back at any part of its course. This can be done with as little trouble as is caused in changing the eye-piece of an ordinary microscope. None of the adjustments of the instrument need be interfered with, and the automatic motion is correct for any number of prisms. A simple contrivance is attached to the micrometer screw, so that it may be thrown out of gear, and the prisms set for any line at pleasure.



With sufficient care the adjustments of the prisms may be made so accurately that any line which falls on the spider-webs of the micrometer eye-piece will appear on the line when any number of the prisms are used. This forms the best method I have yet tried of accurately adjusting a spectroscope of high dispersive power, and enables me to obtain better definitions than I have hitherto done.

In the diagram P P &c. are the ordinary equilateral prisms. R is the reflecting prism shown at the end of the train; in which case, as the ray is reversed, a dispersive power equal to 12 prisms would be obtained. R′ shows the same prism in dotted lines, placed behind the first prism; thus giving the lowest dispersive

power with which the instrument could be used, equal to 2 prisms. D D D represents the dovetails in which the reflecting prism can be made to slide between any of the equilateral prisms.

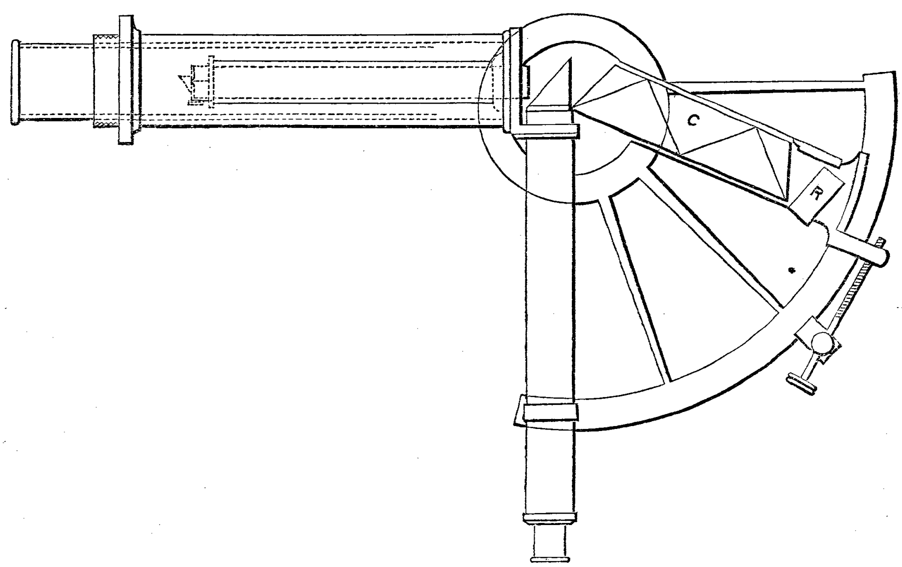
In this instrument both the telescope and collimator remain stationary, so that neither the light nor the eye of the observer need be moved during a whole series of observations. By simply turning the micrometer-screw the train of prisms is moved, so that the whole of the spectrum passes before the eye of the observer, each ray, as it comes to the centre of the field of view, passing through the whole of the prisms at the minimum angle of deviation; thus ensuring the finest definition.

Such a spectroscope, fitted up as a tele-spectroscope, and used with a telescope above 6 in. in aperture, would answer equally well for observing the spectra of the stars, nebulæ, comets, planets, or the Sun, thus saving the multiplication of apparatus, and the great inconvenience of shifting the instruments when they were in adjustment.

While possessing these advantages, this instrument is at the same time more easy to use than any ordinary spectroscope.

On a Tele-Spectroscope for Solar Observations. By John Browning.

The instrument made by me for Mr. Lockyer appears to answer its purpose very well, yet it leaves something to be desired on the score of lightness, and the manner of attaching



it to the eye-piece of the telescope makes it necessarily expensive. I have therefore sought to introduce an instrument which could be used with greater facility. This I have accomplished by

using a Rutherfurd's compound prism, as described in my paper, No. 7, vol. xxxi. of the *Monthly Notices*. The ray is reversed twice, so that the power of this prism is quadrupled. As the instrument is made as light as possible, it can be screwed into the eye-piece of a telescope in the same manner as an ordinary eye-piece, and it is sufficiently light to be used with a telescope of 4 in. in aperture.

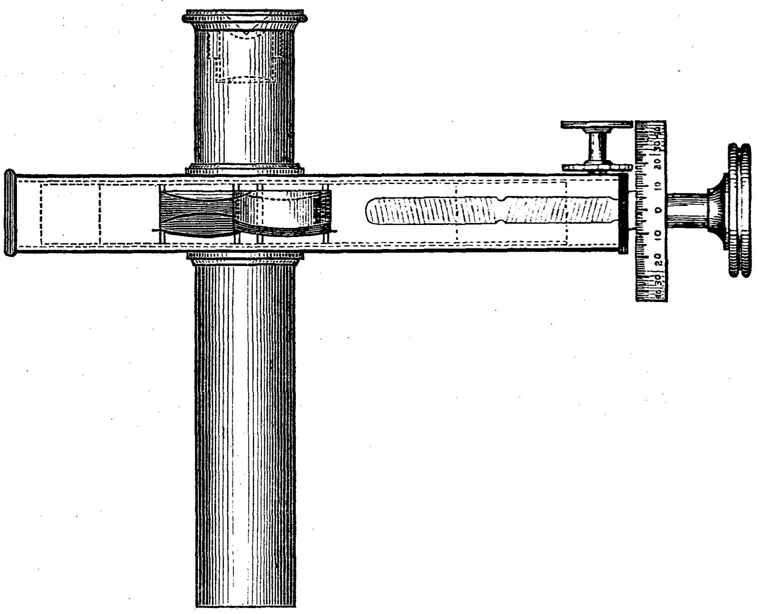
In the diagram C is the compound prism; R the reflecting prism by which the reversion of the ray is accomplished. A second prism of a similar kind is placed directly underneath this, in such a manner that it could not be shown in the diagram.

In using the instrument, it must be so arranged that the image of the Sun falls in focus on the slit.

The dispersive power of the instrument may be varied to suit the aperture of the telescope employed, so as to be equal to between 6 and 12 equilateral prisms of dense flint-glass.

On a Double-Image Micrometer. By John Browning.

As double-image micrometers can be used on a telescope without a clock-driving apparatus, and without the field of view being illuminated, they would doubtless be more employed if there were not important drawbacks to their general use.



Having recently considered the various contrivances used as double-image micrometers, I came to the conclusion that an instrument in many ways more advantageous might be made by simply dividing an ordinary Barlow lens, and causing the two

halves to move separately by means of a micrometer-screw. (See Diagram.) Having had this contrivance made, I find its performance even more satisfactory than I had anticipated. It is compact, inexpensive, and possesses the great advantage that it may be used with any eye-piece.

A Suggestion in the Use of Chronometers, with a view to its use in the approaching Transit of Venus. By David Gill, Jun., F.R.A.S.

Whilst equal increments of heat produce nearly equal effect of retardation on an uncompensated watch, the same increments of heat do not cause the weights of the ordinary chronometer compensation balance to approach the centre of the balance with equal increments.

Besides this, to produce perfect compensation the weights should approach to the centre with an increasing ratio to the temperature, and thus the result is to produce an irrationality of compensation, very analogous to the irrationality of spectra of crown or flint glass, producing secondary spectra in an object-glass.

It is evident from this that if such a balance is adjusted to produce perfect compensation at two very extreme temperatures, it will be over-compensated in mean temperatures.

Many circumstances have been made to correct this, notably by Mr. Hartnup, of Liverpool Observatory, by Messrs. Loseley, Kullberg, and others.

These have been attended with more or less success, but all require very great care in execution and much nicety in adjustment; so that it is still a question if the old form of balance, with certain precautions in its use, is not capable of giving the best results. What I would propose is this, that in the same box with the chronometer should be another chronometer, which we shall call the chronometric thermometer, provided either with an uncompensated brass balance, or a balance where the positions of brass and steel are reversed so as to exaggerate, instead of diminishing, the effects of temperature.

The *true* rate of the chronometer is then to be found for each rate of the chronometric thermometer relative to the chronometer.

Both Greenwich and Liverpool now possess the most perfect means for ascertaining such rates, and could issue them in a tabulated form, with each instrument so tried.

If this were well carried out, and attention also paid to acceleration of mean rate, and allowance to be made for it, as pointed out by Mr. Hartnup, I am convinced that more perfect results than have been hitherto attained by chronometer transport could be arrived at.
